

FUTURE ACCELERATORS PROGRESS MEETING

1st November, 1957.

1. SPIRAL RIDGE CYCLOTRONS1.1 Theory

The theoretical study for conversion of the Harwell synchrocyclotron has now been completed and the results have been published in report form.

Some thought has been given to the possibility of a 600 MeV, completely C.W., machine. There are no fundamental dynamical problems like those in the conversion, and a broad design may be set down in a relatively short time. The least number of ridges which may be considered is $N = 8$, and in order to obtain sufficient focusing at the centre, the design features must be similar to the 900 MeV, $N = 12$, Oak Ridge proposal - that is, open-sector, large radius, low field. The cost may be decreased by reducing the δ demanded, (δ need not necessarily approach unity with open sector), but the field at the centre must then be increased, leading to vertical focusing troubles. A compromise must be reached between these effects. A possible way of overcoming this problem would be to revert to the $N = (4, 8)$ composite system proposed previously for high energy cyclotrons.

Summing up, the problems are more closely connected with magnet design and cost studies than with fundamental particle dynamics.

1.2 Electron Model

The coils have been constructed and out to shape; they are now being Araldited in position. The pulsed r.f. oscillator to drive the dees is complete. The dee and vacuum chamber have been designed and are due shortly. The power supplies and transistor units for providing stabilised currents to the coils, are going forward.

A great deal of field measurement will have to be carried out; the field due to each coil must be measured separately, and the necessary current distribution to give the required field must then be calculated. Measurements will not enable us to do this as accurately as is required, and final trimming must be done with the aid of the beam.

2. SPIRAL RIDGE, OPEN SECTOR, F.F.A.G.2.1 Parameters

The list of parameters, some of which have been reported previously, is given below:

Final Energy	15 BeV
No. of periods/magnet sector	$1\frac{3}{4}$
No. of periods/superperiod	$7 = 4 \times 1\frac{3}{4}$
No. of superperiods	$S = 8$
No. of periods	$N = 8 \times 7 = 56$
No. of straight sections	$(56/1.75) = 32$
Power law index	$k = 74.34$
Length of st. section/length of magnet sector	$L_s/L_m = 0.05$
Spiral Angle,	$\phi = 5.50^\circ$
Mean Field	$B_0 = 7 \text{ kG.}$
Radial Aperture:	
(i) 10 MeV injection	4.78 m.
(ii) 50 MeV injection	3.96 m.
Magnet Sector Radius:	75.8 m.
Magnet Sector length.	$L_M = 14.89 \text{ m.}$
Straight section length	$L_s = 0.744 \text{ m.}$
Spiral Pitch	0.817 m.

2.2 Fine-Structure Resonances

The resonance diagram in terms of μ_V and μ_R is plotted in Fig. 1. The non-linear coupled, $2Q_V = Q_R$, resonance is an even more dangerous instability in these machines than in S/R cyclotrons. Its behaviour with increasing radial betatron amplitude A_R is illustrated in Fig. 2. The ordinate is a measure of Q_V for zero radial oscillation. The dotted lines of Fig. 2 are contours of constant Q_V .

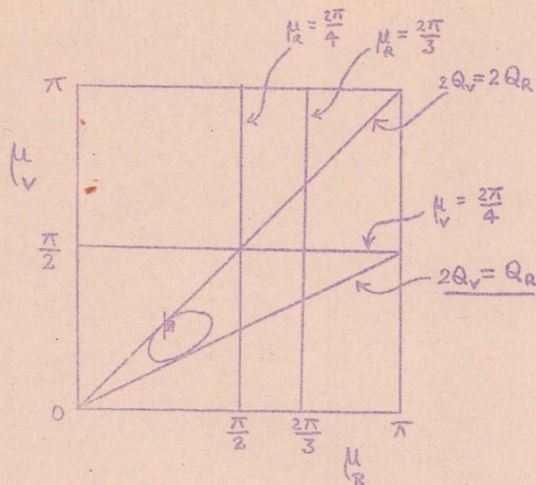


FIG. 1

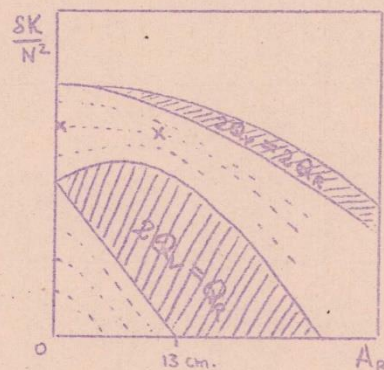


FIG. 2

It will be seen that in the region below the $2Q_V = Q_R$ resonance, Q_V changes violently with A_R , but that between the resonances it does not appear to vary greatly. We therefore select our working point in this region, somewhere within the oval contour on Fig. 1, and nearer to $2Q_V = 2Q_R$ than to $2Q_V = Q_R$.

2.3 Superperiod Resonances

Expanding Fig. 1 in this region, we draw in the superperiod resonances as in Fig. 3.

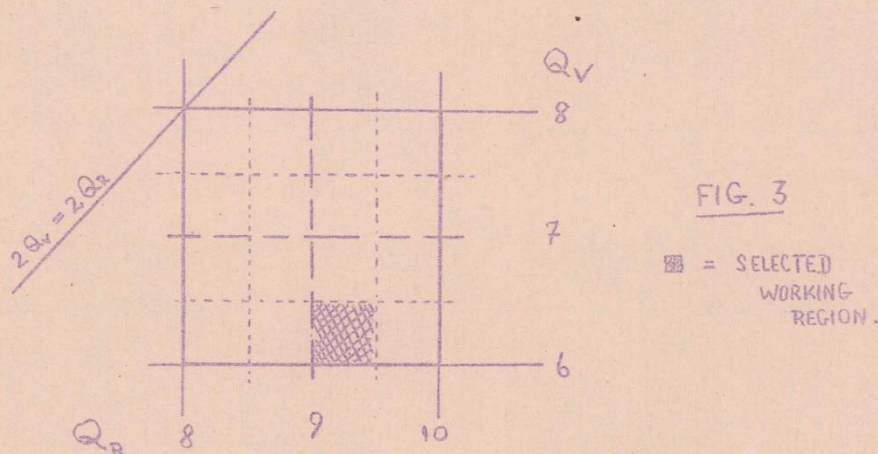


FIG. 3

■ = SELECTED
WORKING
REGION.

We chose $k \approx 75$, so that $Q_R^2 \approx (1+k)$ gives Q_R in the region of 9.

Since $S = 8$

$$\mu_{s.p.} = 8\mu_V = 8 \times \frac{2\pi Q}{8}$$

$$\therefore Q_R = 8, \quad \mu_{s.p.} = 2\pi$$

$$Q_R = 10, \quad \mu_{s.p.} = 2\pi + \frac{2\pi}{4}$$

These are intrinsic resonances, analogous to zero and $2\pi/4$ radial modes for the finer periodic structure. Similarly, intrinsic vertical resonances occur for $Q_V = 10$ and 6. These lines give the large square on Fig. 3. In addition, there will be superperiod integral error resonances for $Q_R = 9$ and $Q_V = 7$, (heavy dotted lines on Fig. 3), and half-integral error resonances further sub-dividing the working region, (fine dotted lines). The net result is that we decide to work in a small $\frac{1}{2} \times \frac{1}{2}$ square, shown hatched on the diagram. The Q -values must not vary by more than $\frac{1}{4}$ if we are to remain inside.

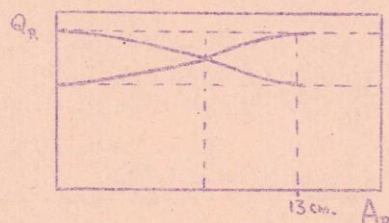
2.4. Tolerances on Shift of Working Point

The effects due to lack of scaling for particles of different momenta have been studied computationally. We denote by φ the phase, $N\phi$, at which a particle of certain momentum enters the ridge structure at the beginning of a magnet sector. Due to our selection of $1\frac{3}{4}$ periods per sector, the maximum change in φ will be between 0 and $\pi/4$. The changes in Q_R and Q_V between these extremities are summarised below:

φ	Q_R	Q_V
0	9.14	6.44
$\pi/4$	9.24	6.35

This is a considerable improvement over the lack of scaling observed in systems studied previously.

Further, the computations give results for the non-linear effects on Q_R , due to the presence of straight sections. That is, the straight sections lead to tilts in the closed orbit and non-linear terms come in. In Fig. 4, we plot the variation in Q_R with increasing radial amplitude, A_R , for the extreme values of φ . The



curves cross over and the initial separation is again reached for $A_R \approx 13$ cm. This is a reasonable amplitude, (e.g. cf. with the CERN machine). Even if there were no straight sections, Q_V would depend on A_R , (see Fig. 2). However if we work in the region $\pi \dots \dots \pi$, Q_V does not appear to change appreciably for $A_R < 13$ cm. It is difficult to be completely certain of this, since the vertical phase plots are more complicated than the radial ones. We go completely round the (Z, Z') phase plots in about 7 superperiods, or 50 periods. Hence, we can only study relatively short-term effects on Q_V ; we consider that new methods of computation must be developed for long-term stability studies.

2.5 R.F. Knockout

Before launching an extensive dynamical programme, it is worth while to re-assess the potential usefulness of this machine. For this purpose we must consider the formidable problem of magnet design, and the no-less crucial question of whether we can get increased mean currents by using Bucket Lift schemes. In this connection, consideration of r.f. knockout has some bearing.

Consider the following scheme. We accelerate protons from some radius r_1 to some new radius r_2 , where we switch off the r.f. and leave the particles circulating. We now return to r_1 , start a new cycle, and bring up another bunch of protons. During this second F.M. cycle, $\omega_1 < \omega < \omega_2$, the frequency differs from the revolution frequency ω_2 of the protons at r_2 . Harmonics of the accelerating field can resonate with the betatron frequency of the protons on r_2 , leading to high-frequency resonance conditions.

Consider a smoothed out radial equation for the stacked protons:

$$\rho'' + Q^2 \rho = f(\theta) \quad \dots\dots\dots (1)$$

Terms on the right hand side could lead to resonance, and these "K.O." terms may arise in different ways. (a) In slanted cavities such as those suggested by Kerst, these are radial components of the r.f., whose harmonics could be operative in causing instability. (b) In our case such components could arise from errors. (c) The intrinsic K.O. terms come from the momentum change. That is, we may write

$$\rho'' + Q^2 \rho = \frac{\Delta p}{p} \sim \exp. i \{ s\theta + nN\theta - \omega_{r.f.} t \} \quad \dots\dots\dots (2)$$

Here, N refers to the number of cavities, and s can take account of how we phase the resonators, i.e. we may take any arbitrary linear phase shift. The harmonic spectrum is illustrated in Fig. 5; by adjusting s we may shift the whole pattern sideways.

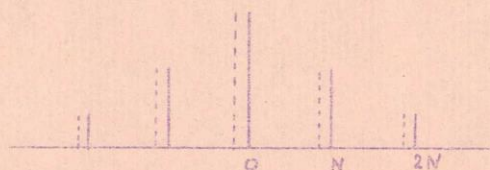


FIG. 5

Now we may write

$$\omega_{r.f.} t = \frac{\omega_{r.f.}}{\dot{\theta}_{st.}} \cdot \theta = s \cdot \frac{\dot{\theta}_p}{\dot{\theta}_{st.}} \cdot \theta \quad \dots\dots\dots (3)$$

where $\dot{\theta}_{st.}$ refers to the stacked protons, and $\dot{\theta}_p$ to the particles coming up in the bucket. Hence K.O. frequencies occur for

$$Q = nN \pm s \left(1 - \frac{\dot{\theta}_p}{\dot{\theta}_{st.}} \right), \text{ or, } \frac{Q}{N} = n \pm \frac{s}{N} \left(1 - \frac{\dot{\theta}_p}{\dot{\theta}_{st.}} \right) \quad (4)$$

With $s = N$ we have 2π -mode acceleration; with $s = \frac{N}{2}$ we have π -mode. These cases are illustrated in Figs. 6(a) and (b), plotting Q/N against $\dot{\theta}_p/\dot{\theta}_{st.}$.

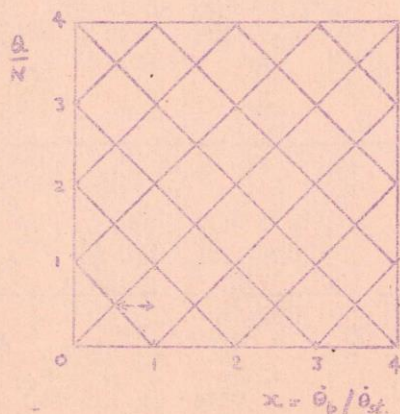


FIG. 6(a) 2π -MODE ACCELERATION

$$\frac{s}{N} = 1, \quad \frac{Q}{N} = n \pm (1-x)$$

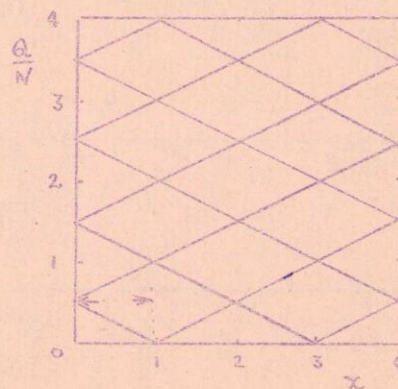


FIG. 6(b) π -MODE ACCELERATION

$$\frac{s}{N} = \frac{1}{2}, \quad \frac{Q}{N} = n \pm \frac{1}{2}(1-x)$$

It will be seen that π -mode gives double the tolerance to avoid K.O. frequencies. Fig. 6(a) shows that we cannot go below half the stacking frequency, but with Fig. 6(b) we can go to zero. We have $Q = 9$ and $N = 32$ in the design.

Errors in phasing or voltages of the cavities can lead to a finer structure inside the working diamond; i.e. we could get a criss-cross series of error lines. However, we can, in principle avoid this by reducing the magnitude of the error.

We can see that if we accelerate on a high harmonic, (high compared with the number of cavities), the K.O. lines rise steeply; we must not use high harmonics.

2.6 Bucket Lift

Several bucket-lift schemes have been proposed by MURA; some of these may be immediately dismissed as impractical. For example, simultaneous acceleration on two different harmonics from a single cavity demands a high harmonic compared with the number of cavities - the situation condemned in the last paragraph. Another difficulty is that the frequency-time curve is rigidly fixed, and we cannot accelerate between two points on it by arbitrarily adjusting the frequency at these points.

The simplest bucket lift scheme may be explained as follows:-

Accelerating a single bunch of particles at constant voltage, the momentum-time curve would be something like curve 1 in Fig. 7. Trying to increase the current, we put in n pulses, (say $n = 100$). We can then only accelerate these to a small fraction of the final energy, - to 100 MeV in a 15 BeV machine. In terms of radial aperture this 100 MeV interval represents about 0.5 m, and we would not like to decrease this, if we are to fit in a reasonable cavity. Now it may not be possible to get 100 stacks at 100 MeV, due to the space-charge limitation on the injected current, (although space-charge neutralisation could help out). So we

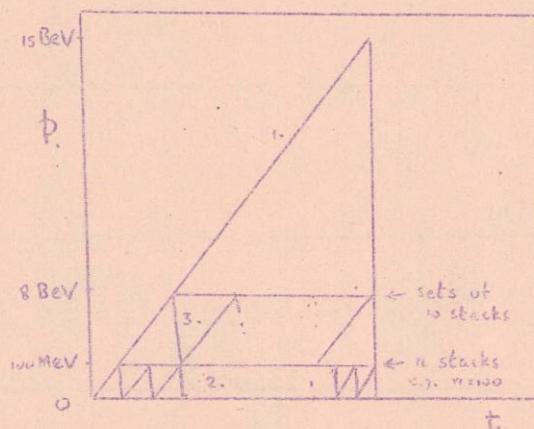


FIG. 7
"BUCKET LIFT."

collect these stacks in sets of 10 and lift them to the ^{next} stage (curve 3). The last $\frac{1}{2}$ m of aperture takes us from 8 BeV to full energy.

We ignore adiabatic damping since it is insignificant until we are near E_{TR} , which is in the region of 9 BeV in our case. Gas scattering effects are also ignored.

Now, consider our first stacks at 100 MeV. Suppose we bring up a bucket, switch off the r.f. to stack it, and return for the next lift.

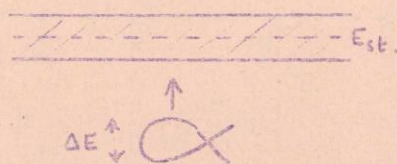


FIG. 8(a)

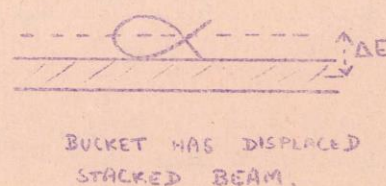


FIG. 8(b)

As we bring the bucket through the stacked particles, we find that it displaces them on the average by about ΔE , the energy width of the bucket. The effect is similar to that where particles slip in phase with respect to a harmonic travelling wave.

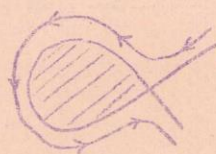
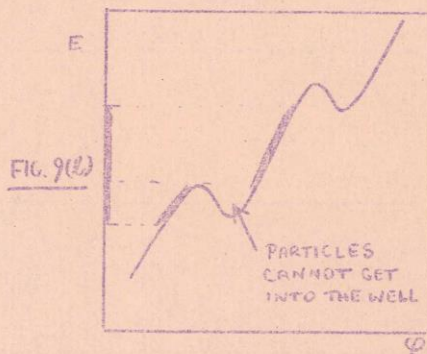


FIG. 9(a)



On the potential well diagram 9(b), we see that they cannot get into the well. The majority slip around the bucket as in 9(a); some may come to rest on the positive part of the wave and are accelerated.

Thus, we obtain a roughly correct picture by taking the energy width of n stacked beams to be $n \Delta E$. Now $\Delta E \sim V_1^2$, so that if we are to trap the n buckets in one large bucket for the next stage of acceleration, we must have $V_2^2 = n V_1^2$, where V_1 and V_2 are the voltages in the first and second stages. The time for acceleration to full energy E_2 at peak voltage V_2 is given by $t_2 \sim E_2/V_2$; the time for acceleration of one bunch to the intermediate energy E_1 is $t_1 \sim E_1/V_1$.

$$\therefore t_2/t_1 = \frac{E_2 V_1}{E_1 V_2} = \frac{E_2}{E_1} \cdot \frac{1}{n}$$

If we are to stack n bunches, then $E_2/E_1 = n^3$. As we have seen, we must have $E_2/E_1 \sim 150$, so that $n \approx 5$. The reduced energy gain per turn in the first stage must be obtained by reducing the volts in the cavities, and by reducing the number of cavities, since the second method leads to a high harmonic mode and greater possibility of knockout.

Another system has been considered, involving 10 independent r.f. oscillators. The ratio is reduced to n^2 for this case, so that $n \approx 10$. Thus we would only gain a factor 2 with a great increase in complexity. In addition this system increases the number of knockout frequencies.

The MIRA calculations suppose that for an energy spread ΔE_{inj} at injection, the phase area at stacking is n times ΔE_{inj} for n buckets. This is only correct if the switching on is adiabatic, since, in a sudden switch-on, the effective ΔE is the bucket width, likely to be much greater than ΔE_{inj} . We may, however, get over this problem by increasing the volts adiabatically, and perhaps stacking adiabatically. This might give us a further factor 2. Our present estimate is, therefore, that bucket lifts might give an increase of a factor 20 in mean intensity; a factor 100, or even 50, is not envisaged.

Several experiments on bucket lift have been carried out. At MIRA, model measurements were made in which the initial acceleration was carried out by betatron action; the r.f. was then switched on and buckets sent up. Nothing was obtained in the first bucket, but subsequent buckets accelerated particles. This may be explained by the downward displacement of the first particles by the later buckets; some protons were displaced sufficiently to be picked up. The experiments verified that the arguments on average downward phase displacement are, in general, true.

Work on the Moscow cyclotron has also been reported. It is stated that, (presumably by cutting off volts at the end of the frequency cycle) 3 or 4 bunches were stacked; on the next cycle, they were carried to full energy with 3-4 times the previous intensity. This does not seem possible on the arguments presented above, and indeed, similar experiments recently carried out at Berkeley confirm our prediction. It is difficult to envisage the mechanism by which the Russians achieved their gain in intensity.

The most advantageous aspect of the beam stacking seems to us to be the possibility of stacking at full energy. With 15 BeV particles, the lifetime seems sufficiently long to build up 100 or more times the circulating current. We wish to assess the advantages of such a machine from a nuclear physics point of view.

2.7 Magnet Problem

Fig. 10 is a nearly-to-scale plan view of a magnet sector.

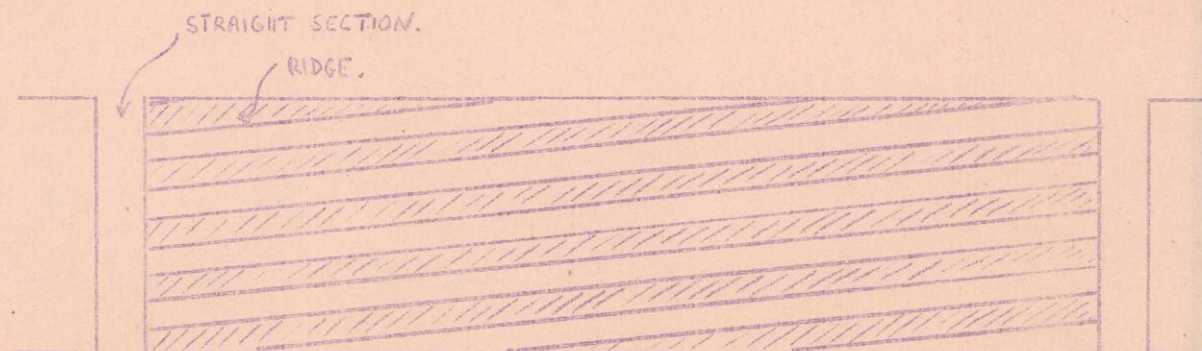
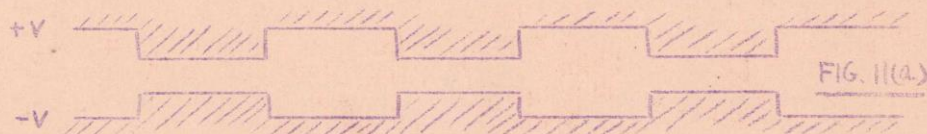


FIG. 10

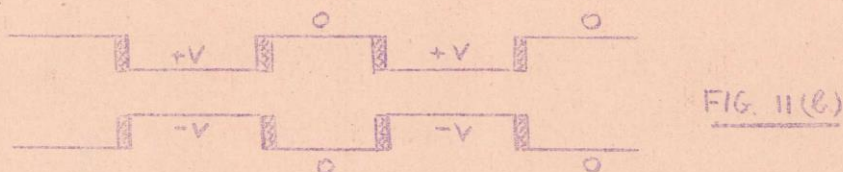
Providing yokes for all of the iron ridges is obviously out of the question; they would all have to be carried around the back of the sector.

Some experimental work will be carried out by the electron model group on poleface windings, using a resistance net analogue.

Measurements have already been done by the magnet group on ridged polefaces.



If the whole field is supplied by poleface windings the potential distribution will look more like Fig. 11(b), and higher values of β will be obtained.



A somewhat academic study will be made, first assuming infinite current density in the windings. When the results of this have been assessed, more specific and realistic arrangements suitable for, (e.g.) open sector cyclotrons will be studied.

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